5-2 Ratios, Rates, and Unit Rates

Learn to work with rates and ratios.

Density is a ratio that compares mass and volume. Different substances have different densities. For example, gold has a density of $\frac{19,300 \text{ kg}}{1 \text{ m}^3}$, or 19,300 kilograms per cubic meter.

The Excentrique MP-400 MP3 player is made of 24-carat gold.

Vocabulary

rate

unit rate

unit price

A **rate** is a comparison of two quantities that have different units.

ratio: $\frac{90}{3}$ rate: $\frac{90 \text{ miles}}{3 \text{ hours}}$ ← *Read as "90 miles per 3 hours."*

Unit rates are rates in which the second quantity is 1. The ratio $\frac{90}{3}$ can be simplified by dividing: $\frac{90}{3} = \frac{30}{1}$.

Interactivities Online ▶

unit rate: $\frac{30 \text{ miles}}{1 \text{ hour}}$, or 30 mi/h

E X A M P L E **1** **Finding Unit Rates**

Miki can type 120 words in 3 minutes. How many words can she type per minute?

$\frac{120 \text{ words}}{3 \text{ minutes}}$ *Write the rate.*

$\frac{120 \text{ words} \div 3}{3 \text{ minutes} \div 3} = \frac{40 \text{ words}}{1 \text{ minute}}$ *Divide to find words per minute.*

Miki can type 40 words in one minute.

Since density is measured in units of mass per unit of volume, it is a unit rate.

E X A M P L E **2** *Chemistry Application*

Reading Math

Units that result from dividing two different units, such as kg/m³, are called *derived units*.

A **Four cubic meters of silver has a mass of 41,960 kilograms. What is the density of silver?**

$\frac{41,960 \text{ kg}}{4 \text{ m}^3}$ *Write the rate.*

$\frac{41,960 \text{ kg} \div 4}{4 \text{ m}^3 \div 4}$ *Divide to find kilograms per 1 m³.*

$\frac{10,490 \text{ kg}}{1 \text{ m}^3}$

Silver has a density of 10,490 kg/m³.

B **Aluminum weighing 1350 kilograms has a volume of 0.5 cubic meters. What is the density of aluminum?**

$\frac{1350 \text{ kg}}{0.5 \text{ m}^3}$ *Write the rate.*

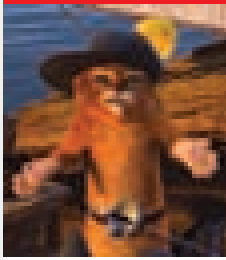$\frac{1350 \text{ kg} \cdot 2}{0.5 \text{ m}^3 \cdot 2}$ *Multiply to find kilograms per 1 m³.*

$\frac{2700 \text{ kg}}{1 \text{ m}^3}$

Aluminum has a density of 2700 kg/m³.

Estimation Estimate each unit rate.

15. 250 heartbeats in 6 minutes

16. $107 for 22 magazines

17. 295 words in 6 minutes

18. 17 apples weigh 4 pounds

19. Multi-Step Before 1986, a gold bullion in the Federal Reserve Bank was rectangular and had a volume of approximately 727.7 cm^3. The density of gold is 19.3 g/cm^3. A pound is approximately 454 g. Find the weight of one gold bullion to the nearest tenth of a pound.

20. Entertainment Tom, Cherise, and Tina work as film animators. The table shows the number of frames each rendered in an 8-hour day.

Frames Rendered	
Name	Frames
Tom	203
Cherise	216
Tina	227

   a. Find the hourly unit rendering rate for each employee.

   b. Who was the most efficient?

   c. How many more frames per hour did Cherise render than Tom?

   d. How many more frames per hour did Tom and Cherise together render than Tina?

21. What's the Error? A clothing store charges $25 for 4 T-shirts. A student says that the unit price is $0.16 per T-shirt. What is the error? What is the correct unit price?

22. Write About It Explain how to find unit rates. Give an example, and explain how consumers can use unit rates to save money.

23. Challenge The size of a television (13 in., 25 in., 32 in., and so on) represents the length of the diagonal of the television screen. An aspect ratio describes a screen by comparing its width to its height. A 25 in. television has an aspect ratio of 4:3. What are the width and height of the screen?

Test Prep and Spiral Review

24. Multiple Choice A 24 lb bag of dog food sells for $10.56. What is the unit price per pound?

   (A) $0.44/lb   (B) $0.53/lb   (C) $13.44/lb   (D) $34.56/lb

25. Extended Response Flowers can be purchased in bunches of 4 for $2.48 or 6 for $3.96. Which is the better buy? Explain.

Solve. (Lesson 1-8)

26. $p - 8 = 12$ **27.** $y + 9 = 15$ **28.** $w - 7 = 8$ **29.** $k + 4 = 11$

Find two ratios that are equivalent to each given ratio. (Lesson 5-1)

30. $\frac{3}{5}$ **31.** $\frac{13}{26}$ **32.** $\frac{4}{11}$ **33.** $\frac{10}{9}$

GUIDED PRACTICE

See Example **1**   **1.** Ana Maria's heart beats 225 times in 3 minutes. How many times does her heart beat per minute?

See Example **2**   **2.** A nickel has a mass of 5 g and a volume of approximately 0.689 cm^3. What is the approximate density of a nickel?

See Example **3**   **3.** A bicyclist rides the first 60 kilometers of a 220-kilometer race in 1.5 hours.

      a. What is the bicyclist's average speed?

      b. A reporter estimates that at this speed the bicyclist will finish the entire race in less than 6 hours. Is the reporter's estimate reasonable? Explain.

See Example **4**   **4.** A 16 oz box of crackers costs $3.99 and a 38 oz box of crackers costs $6.99. Which is the better buy?

INDEPENDENT PRACTICE

See Example **1**   **5.** Kenji earns $32 in 4 hours. How much does he earn per hour?

See Example **2**   **6.** The mass of a diamond is 1.76 g. The volume is 0.5 cm^3. What is the density of the diamond?

See Example **3**   **7.** In 1860 and 1861, riders of the Pony Express delivered mail on horseback from St. Joseph, Missouri, to Sacramento, California. The trip of 1966 miles took about 10 days (240 hours).

      a. What was the riders' average speed to the nearest tenth of a mile per hour?

      b. A student estimates that it would take a Pony Express rider less than 6 hours to travel 75 miles. Is the student's estimate reasonable? Explain.

See Example **4**   **8.** One yard of ribbon costs $0.49 and 3 yards of ribbon costs $1.49. Which is the better buy?

      9. A 16 oz package of brown rice costs $0.79 and a 32 oz package of brown rice costs $3.49. Which is the better buy?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP10.

Find each unit rate.

10. travel 804 miles in 16 hours   **11.** score 84 points in 6 games

12. $7.05 for 3 tacos   **13.** 64 beats in 4 measures of music

14. Social Studies *Population density* is a unit rate comparing population to area. The nation of Malta has an area of 122 square miles and a population of about 402,000. The U.S. has an area of 3,794,083 square miles and a population of about 301,140,000. What is the approximate population density of Malta? of the U.S.?

Average rate of speed is the ratio of distance traveled to time. This relationship can be expressed by the formula $r = \frac{d}{t}$, where r is the average rate of speed, d is distance, and t is time.

EXAMPLE **3** *Travel Application*

A band's tour bus is traveling 525 miles from Charlotte, North Carolina, to Orlando, Florida.

A In the first 5 hours of the trip, the bus travels 260 miles. What is the bus's average speed?

$$r = \frac{d}{t}$$ *Find the ratio of distance to time.*

$$= \frac{260 \text{ mi}}{5 \text{ h}}$$ *Substitute 260 miles for d and 5 hours for t.*

$$= 52 \text{ mi/h}$$ *Divide to find the unit rate.*

The bus's average speed is 52 mi/h.

B The driver estimates that the entire trip will take 8 hours. If the bus keeps traveling at the same average speed, is the driver's estimate reasonable? Explain.

Determine how long the trip will take.

$$d = rt$$ *Use the formula d = rt.*

$$525 = 52t$$ *Substitute 525 for d and 52 for r.*

$$\frac{525}{52} = \frac{52t}{52}$$ *Divide both sides by 52.*

$$10.1 \approx t$$ *Simplify.*

At an average speed of 52 mi/h, the trip will take about 10 hours. The driver's estimate is not reasonable.

Helpful Hint

The formula $r = \frac{d}{t}$ is equivalent to $d = rt$, as shown below.

$$r = \frac{d}{t}$$
$$r \cdot t = \frac{d}{t} \cdot t$$
$$rt = d$$

Unit price is a unit rate used to compare price per item.

EXAMPLE **4** **Finding Unit Prices to Compare Costs**

Arnie can buy a 16 oz box of cereal for $5.49 or a 20 oz box for $5.99. Which is the better buy?

$$\frac{\text{price for box}}{\text{number of ounces}} = \frac{\$5.49}{16 \text{ oz}} \approx \$0.34/\text{oz}$$

$$\frac{\text{price for box}}{\text{number of ounces}} = \frac{\$5.99}{20 \text{ oz}} \approx \$0.30/\text{oz}$$

Divide the price by the number of ounces.

The better buy is the 20 oz box for $5.99.

Think and Discuss

1. Choose the quantity that has a lower unit price: 6 oz for $1.29 or 15 oz for $3.00. Explain your answer.