Algebra IIXA class notes 5.1

Using Transformations to Graph Quadratic Functions

Warm Up

For each translation of the point (–2, 5), give the coordinates of the translated point.

1. 6 units down

2. 3 units right

For each function, evaluate $f(-2)$, $f(0)$, and $f(3)$.

3. $f(x) = x^2 + 2x + 6$

4. $f(x) = 2x^2 - 5x + 1$

Objectives

Transform quadratic functions.

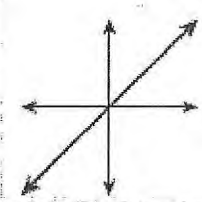
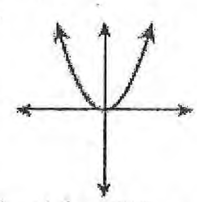
Describe the effects of changes in the coefficients of $y = a(x - h)^2 + k$.

Vocabulary

quadratic function, parabola, vertex of a parabola & vertex form

In Chapters 2 and 3, you studied linear functions of the form $f(x) = mx + b$. A <u>quadratic function</u> is a function that can be written in the form of $f(x) = a\,(x - h)^2 + k$ $(a \neq 0)$. In a quadratic function, the variable is always squared. The table shows the linear and quadratic parent functions.

Linear and Quadratic Parent Functions

<table>
<tr><th>ALGEBRA</th><th>NUMBERS</th><th>GRAPH</th></tr>
<tr><td>Linear Parent Function
$f(x) = x$</td><td><table><tr><td>x</td><td>−2</td><td>−1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$f(x) = x$</td><td>−2</td><td>−1</td><td>0</td><td>1</td><td>2</td></tr></table></td><td></td></tr>
<tr><td>Quadratic Parent Function
$f(x) = x^2$</td><td><table><tr><td>x</td><td>−2</td><td>−1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$f(x) = x^2$</td><td>4</td><td>1</td><td>0</td><td>1</td><td>4</td></tr></table></td><td></td></tr>
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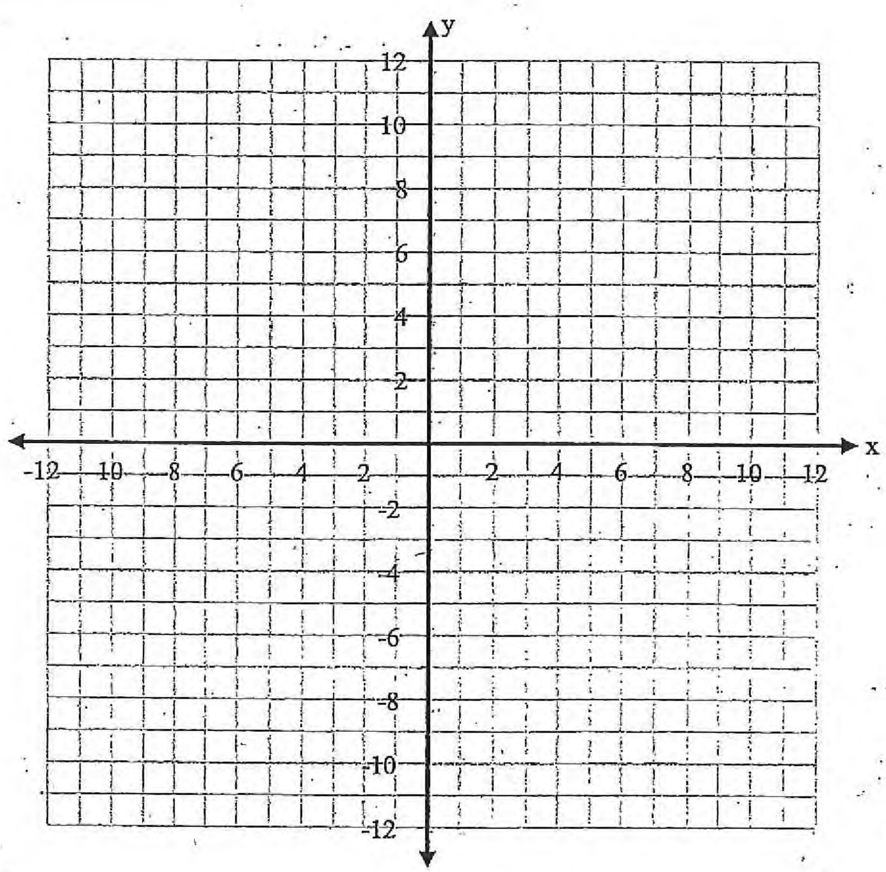
Notice that the graph of the parent function $f(x) = x^2$ is a U-shaped curve called a <u>parabola</u>. As with other functions, you can graph a quadratic function by plotting points with coordinates that make the equation true.

Example 1: Graphing Quadratic Functions Using a Table

A) Graph $f(x) = x^2 - 4x + 3$ by using a table.

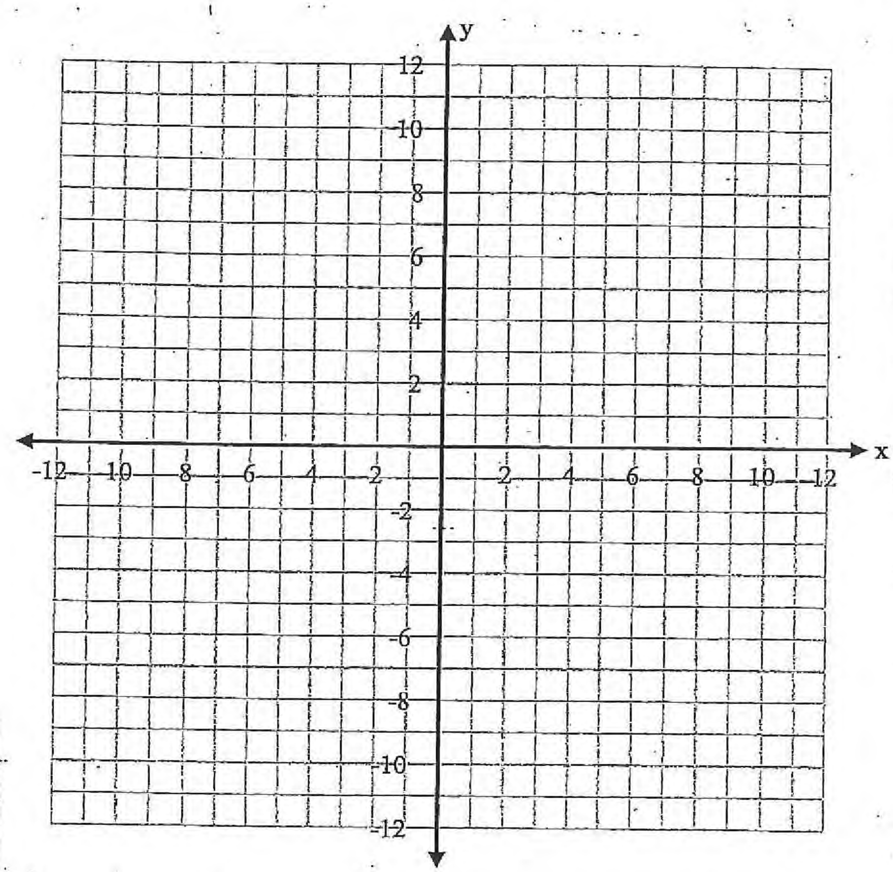
Make a table. Plot enough ordered pairs to see both sides of the curve.

x	$f(x) = x^2 - 4x + 3$	$(x, f(x))$
0	$f(0)=$	
1	$f(1)=$	
2	$f(2)=$	
3	$f(3)=$	
4	$f(4)=$	

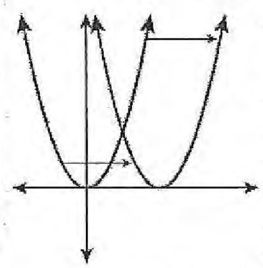
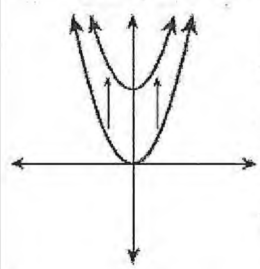


B) Graph $g(x) = -x^2 + 6x - 8$ by using a table.

x	$f(x) = -x^2 + 6x - 8$	$(x, f(x))$
0	$f(0)=$	
1	$f(1)=$	
2	$f(2)=$	
3	$f(3)=$	
4	$f(4)=$	



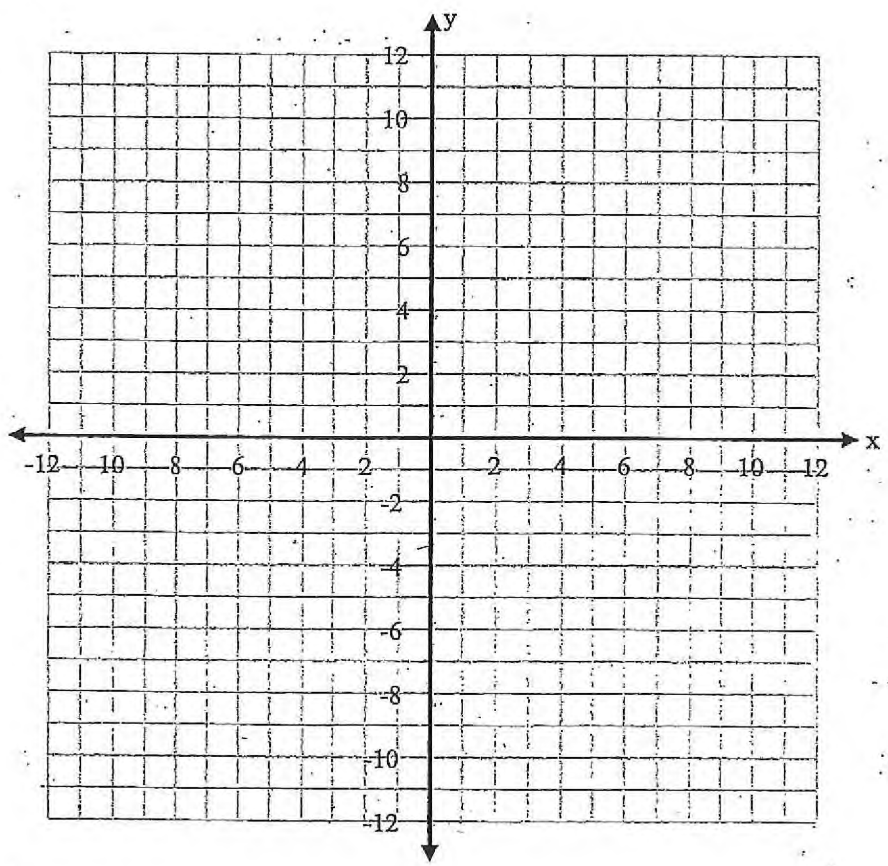
You can also graph quadratic functions by applying transformations to the parent function $f(x) = x^2$. Transforming quadratic functions is similar to transforming linear functions (Lesson 2-6).

Translations of Quadratic Functions	
Horizontal Translations	**Vertical Translations**
Horizontal Shift of $\lvert h \rvert$ Units	Vertical Shift of $\lvert k \rvert$ Units
 $f(x) = x^2$ $f(x - h) = (x - h)^2$ Moves left for $h < 0$ Moves right for $h > 0$	 $f(x) = x^2$ $f(x) + k = x^2 + k$ Moves down for $k < 0$ Moves up for $k > 0$

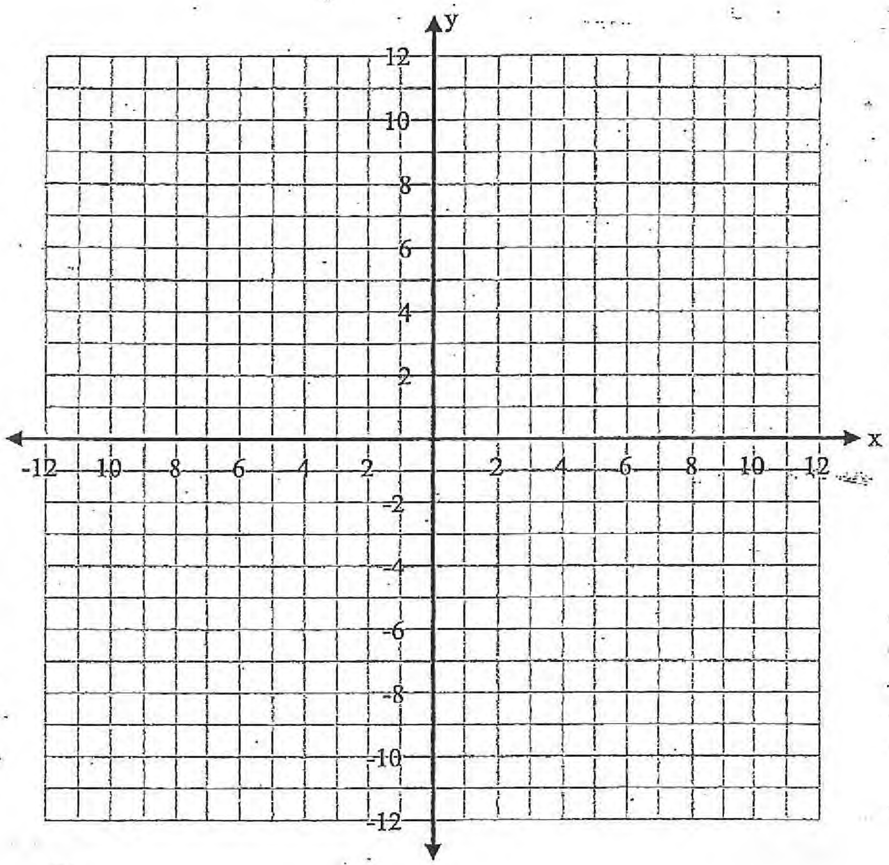
Example 2A: Translating Quadratic Functions

Use the graph of $f(x) = x^2$ as a guide, describe the transformations and then graph each function.

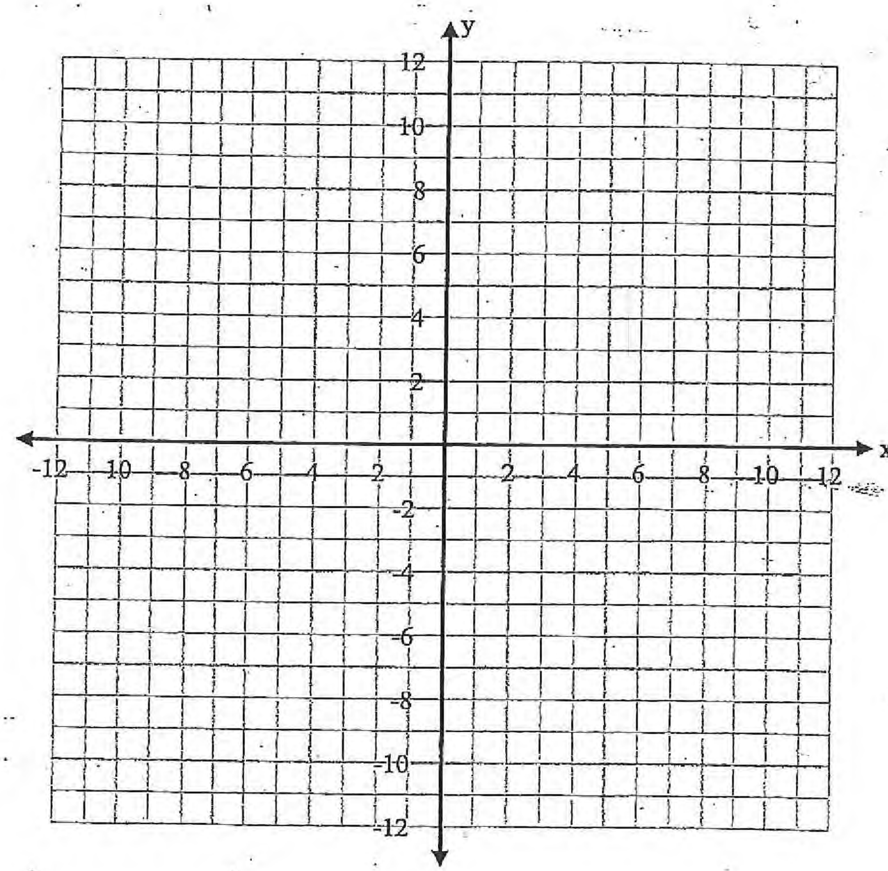
A) $g(x) = (x - 2)^2 + 4$



$B)\ g(x) = (x+2)^2 - 3$



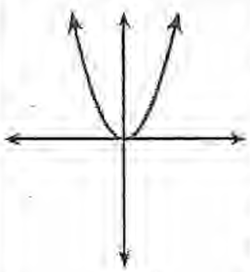
$C)\ g(x) = x^2 - 5$



Recall that functions can also be reflected, stretched, or compressed.

Reflections

Reflection Across *y*-axis



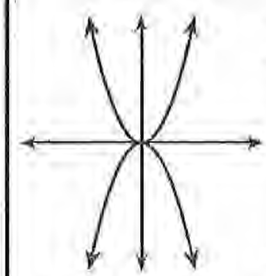
Input values change.

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

The function $f(x) = x^2$ is its own reflection across the *y*-axis.

Reflection Across *x*-axis



Output values change.

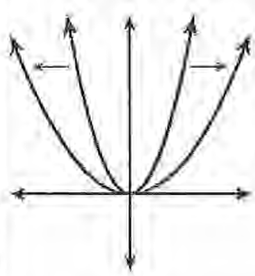
$f(x) = x^2$

$-f(x) = -(x^2)$
$= -x^2$

The function is flipped across the *x*-axis.

Stretches and Compressions

Horizontal Stretch/Compression by a Factor of $|b|$



Input values change.

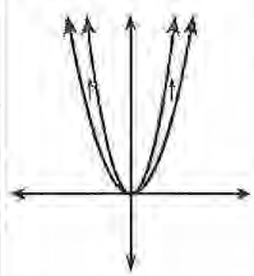
$f(x) = x^2$

$f\left(\frac{1}{b}x\right) = \left(\frac{1}{b}x\right)^2$

$|b| > 1$ stretches away from the *y*-axis.

$0 < |b| < 1$ compresses toward the *y*-axis.

Vertical Stretch/Compression by a Factor of $|a|$



Output values change.

$f(x) = x^2$

$a \cdot f(x) = ax^2$

$|a| > 1$ stretches away from the *x*-axis.

$0 < |a| < 1$ compresses toward the *x*-axis.

Example 3A: Reflecting, Stretching, and Compressing Quadratic Functions

Using the graph of $f(x) = x^2$ as a guide, describe the transformations and then graph each function.

A) $g(x) = -0.25x^2$

B) $g(x) = (3x)^2$

C) $g(x) = (2x)^2$

D) $g(x) = -0.5x^2$

Homework: Read Section 5-1 Take Notes from the reading (T,D,P's)

Complete exercises #1-13 & 17 – 28 pgs. 320