by Graphing

Objectives
Identify solutions of systems of linear equations in two variables.

Solve systems of linear equations in two variables by graphing.

Vocabulary
system of linear equations
solution of a system of linear equations

Why learn this?

You can compare costs by graphing a system of linear equations. (See Example 3.)

Sometimes there are different charges for the same service or product at different places. For example, Bowl-o-Rama charges $2.50 per game plus $2 for shoe rental while Bowling Pinz charges $2 per game plus $4 for shoe rental. A *system of linear equations* can be used to compare these charges.

A **system of linear equations** is a set of two or more linear equations containing two or more variables. A **solution of a system of linear equations** with two variables is an ordered pair that satisfies each equation in the system. So, if an ordered pair is a solution, it will make both equations true.

EXAMPLE 1 Identifying Solutions of Systems

Tell whether the ordered pair is a solution of the given system.

A $(4, 1); \begin{cases} x + 2y = 6 \\ x - y = 3 \end{cases}$

$x + 2y = 6$	
$(4) + 2(1)$	6
$4 + 2$	6
6	6 ✓

$x - y = 3$	
$(4) - (1)$	3
3	3 ✓

Substitute 4 for x and 1 for y in each equation in the system.

The ordered pair $(4, 1)$ makes both equations true.

$(4, 1)$ is a solution of the system.

Helpful Hint

If an ordered pair does not satisfy the first equation in the system, there is no need to check the other equations.

B $(-1, 2); \begin{cases} 2x + 5y = 8 \\ 3x - 2y = 5 \end{cases}$

$2x + 5y = 8$	
$2(-1) + 5(2)$	8
$-2 + 10$	8
8	8 ✓

$3x - 2y = 5$	
$3(-1) - 2(2)$	5
$-3 - 4$	5
-7	5 ✗

Substitute −1 for x and 2 for y in each equation in the system.

The ordered pair $(-1, 2)$ makes one equation true, but not the other.

$(-1, 2)$ is not a solution of the system.

CHECK IT OUT! **Tell whether the ordered pair is a solution of the given system.**

1a. $(1, 3); \begin{cases} 2x + y = 5 \\ -2x + y = 1 \end{cases}$ yes

1b. $(2, -1); \begin{cases} x - 2y = 4 \\ 3x + y = 6 \end{cases}$ no

1 Introduce

EXPLORATION

6-1 Solving Systems by Graphing

You will need a graphing calculator for this Exploration.

1. Press [Y=] and enter the equations $y = 2x - 5$ and $y = -x + 4$ as Y1 and Y2.
2. Press [2nd] [GRAPH] to view a table of values for the two equations.
3. Use the table to find an x-value that produces the same y-value for both equations. Write this x-value and the corresponding y-value as an ordered pair.
4. Use the arrow keys to scroll up and down the table. Does there appear to be any other x-value that produces the same y-value for both equations?
5. Press [GRAPH] to view a graph of the equations.

THINK AND DISCUSS

6. Describe the graph of the functions.
7. Explain what happens on the graph at the point that you found in Step 3.

Motivate

Have students choose an ordered pair that is a solution of $y = x + 1$. Point out that there are infinitely many ordered-pair solutions. Then ask which of the ordered pairs is also a solution of $y = 5 - x$. Let them try several pairs. Tell students that there is only one ordered pair that is a solution of both equations. $(2, 3)$ Then explain that this lesson will show a method for finding that pair.

Explorations and answers are provided in the *Explorations* binder.

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Identify solutions of systems of linear equations in two variables.

Solve systems of linear equations in two variables by graphing.

PREMIER Online Edition
Tutorial Videos, Interactivity, TechKeys, Graphing Calculator

Countdown to Testing Week 13

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Warm Up

Evaluate each expression for $x = 1$ and $y = -3$.

1. $x - 4y$ 13
2. $-2x + y$ −5

Write each equation in slope-intercept form.

3. $y - x = 1$ $y = x + 1$
4. $2x + 3y = 6$ $y = -\frac{2}{3}x + 2$
5. $0 = 5y + 5x$ $y = -x$

Also available on transparency

Math Humor

Teacher: What is the name of the formula that describes the phases of the moon?

Student: The lunear (linear) equation

State Resources

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Additional Examples

Example 1

Tell whether the ordered pair is a solution of the given system.

A. $(5, 2)$; $\begin{cases} \frac{2}{5}x - y = 0 \\ 3x - y = 13 \end{cases}$ yes

B. $(-2, 2)$; $\begin{cases} x + 3y = 4 \\ -x + y = 2 \end{cases}$ no

Example 2

Solve each system by graphing.

A. $\begin{cases} y = x \\ y = -2x - 3 \end{cases}$ $(-1, -1)$

B. $\begin{cases} y = x - 6 \\ y + \frac{1}{3}x = -1 \end{cases}$ $\left(\frac{15}{4}, -\frac{9}{4}\right)$

Also available on transparency

INTERVENTION
Questioning Strategies

EXAMPLE 1

- How do you know when an ordered pair is a solution of a system of linear equations?

EXAMPLE 2

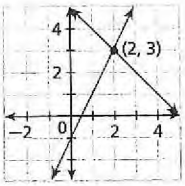
- What does the intersection of the two lines represent?

Teaching Tip **Technology** In **Example 2B,** make sure that students know how to find the point of intersection on a graphing calculator.

After both lines are graphed, press 2nd TRACE and then choose 5.

All solutions of a linear equation are on its graph. To find a solution of a system of linear equations, you need a point that each line has in common. In other words, you need their point of intersection.

$\begin{cases} y = 2x - 1 \\ y = -x + 5 \end{cases}$



The point $(2, 3)$ is where the two lines intersect and is a solution of both equations, so $(2, 3)$ is the solution of the system.

EXAMPLE 2 Solving a System of Linear Equations by Graphing

Solve each system by graphing. Check your answer.

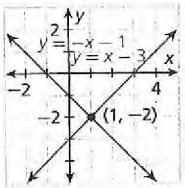
Helpful Hint

Sometimes it is difficult to tell exactly where the lines cross when you solve by graphing. It is good to confirm your answer by substituting it into both equations.

A $\begin{cases} y = x - 3 \\ y = -x - 1 \end{cases}$

Graph the system.

The solution appears to be at $(1, -2)$.



Check

Substitute $(1, -2)$ into the system.

$y = x - 3$	
(-2)	$(1) - 3$
-2	-2 ✓

$y = -x - 1$	
(-2)	$-(1) - 1$
-2	-2 ✓

$(1, -2)$ is a solution of the system.

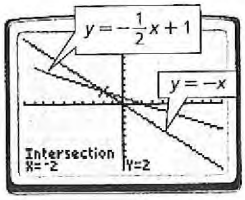
B $\begin{cases} x + y = 0 \\ y = -\frac{1}{2}x + 1 \end{cases}$

$x + y = 0$ *Rewrite the first equation in slope-intercept form.*

$-x \quad -x$

$y = -x$

Graph using a calculator and then use the intersection command.



Check Substitute $(-2, 2)$ into the system.

$x + y = 0$	
$(-2) + (2)$	0
0	0 ✓

$y = -\frac{1}{2}x + 1$	
(2)	$-\frac{1}{2}(-2) + 1$
2	$1 + 1$
2	2 ✓

The solution is $(-2, 2)$.

CHECK IT OUT! **Solve each system by graphing. Check your answer.**

2a. $\begin{cases} y = -2x - 1 \\ y = x + 5 \end{cases}$ $(-2, 3)$ **2b.** $\begin{cases} y = \frac{1}{3}x - 3 \\ 2x + y = 4 \end{cases}$ $(3, -2)$

2 Teach

Guided Instruction

Before teaching students to solve a system by graphing, review slope-intercept form and how to write an equation in that form.

Teaching Tip **Communicating Math** Warn students that the brace, {, is not always used to indicate a system of equations. A standardized test might omit the symbol and simply say that the equations are a system.

Reaching All Learners
Through Multiple Representations

Have students copy and complete the table for the system of equations.

$\begin{cases} y = -2x + 9 \\ y = x + 3 \end{cases}$

x	$y_1 = -2x + 9$	$y_2 = x + 3$
0	9	3
1	7	4

Graph the system, and discuss similarities between the table and the graph. For example, from the table, you can see that when $x = 0$, the y-value of one equation is 9, and the y-value of the other equation is 3. On the graph you can see that the point $(0, 9)$ is on one line and the point $(0, 3)$ is on the other line.

SOLVING

the cost to bowl be the same at both places? What is that cost?

1 Understand the Problem

The answer will be the number of games played for which the total cost is the same at both bowling alleys. **List the important information:**

- Game price: Bowl-o-Rama $2.50 Bowling Pinz: $2
- Shoe-rental fee: Bowl-o-Rama $2 Bowling Pinz: $4

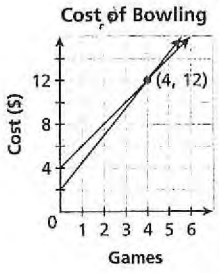
2 Make a Plan

Write a system of equations, one equation to represent the price at each company. Let x be the number of games played and y be the total cost.

	Total cost	is	price per game	times	games	plus	shoe rental.
Bowl-o-Rama	y	=	2.5	•	x	+	2
Bowling Pinz	y	=	2	•	x	+	4

3 Solve

Graph $y = 2.5x + 2$ and $y = 2x + 4$. The lines appear to intersect at $(4, 12)$. So, the cost at both places will be the same for 4 games bowled and that cost will be $12.



4 Look Back

Check $(4, 12)$ using both equations.

Cost of bowling 4 games at Bowl-o-Rama:
$\$2.5(4) + \$2 = 10 + 2 = 12$ ✓

Cost of bowling 4 games at Bowling Pinz:
$\$2(4) + \$4 = 8 + 4 = 12$ ✓

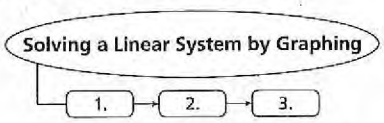
CHECK IT OUT!

3. Video club A charges $10 for membership and $3 per movie rental. Video club B charges $15 for membership and $2 per movie rental. For how many movie rentals will the cost be the same at both video clubs? What is that cost? 5 movies; $25

THINK AND DISCUSS

1. Explain how to use a graph to solve a system of linear equations.
2. Explain how to check a solution of a system of linear equations.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write a step for solving a linear system by graphing. More boxes may be added.

Know it! Note



Students sometimes have difficulty correctly assigning x and y variables in word problems, such as in **Example 3.** Remind them that the value of y cannot be determined unless the value of x is known. In other words, y is dependent on x.

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Additional Examples

Example 3

Wren and Jenni are reading the same book. Wren is on page 14 and reads 2 pages every night. Jenni is on page 6 and reads 3 pages every night. After how many nights will they have read the same number of pages? 8 nights How many pages will that be? 30 pages

INTERVENTION
Questioning Strategies

EXAMPLE 3

- What does the graph show on the left side of the point of intersection?
- What does the graph show on the right side of the point of intersection?

Teaching Tip

Visual Cues In **Example 3** students may benefit from using a table to organize the information.

	Shoe Rental	Game Price
Bowl-o-Rama	$2.00	$2.50
Bowling Pinz	$4.00	$2.00

3 Close

Summarize

Have students graph the equations below to determine whether each ordered pair is the solution of the given system. Have them check their work by substituting the ordered pair into the system of equations.

1. $(0, -2)$ $\begin{cases} y = x - 2 \\ 4y + \frac{1}{2}x = -8 \end{cases}$ yes

2. $(3, 6)$ $\begin{cases} y + 3x = 9 \\ y = 2x \end{cases}$ no

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
6-1 Warm Up, TE p. 383

***Monitor During* the Lesson**
Check It Out! Exercises, SE pp. 383–385
Questioning Strategies, TE pp. 384–385

***Assess After* the Lesson**
6-1 Lesson Quiz, TE p. 388
Alternative Assessment, TE p. 388

Answers to *Think and Discuss*

1. Locate the point of intersection. The ordered pair is a solution of the system.
2. Substitute the x- and y-values of the ordered pair. If both equations are true, the solution is correct.
3. See p. A6.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–3

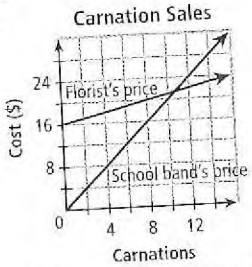
Basic 9–22, 28–30, 33–42
Average 9–16, 18–22, 26–42
Advanced 9–16, 18–42

Homework Quick Check
Quickly check key concepts.
Exercises: 10, 14, 16, 18, 22

Answers

17a. $\begin{cases} y = 2x \\ y = 16 + 0.50x \end{cases}$

b.



It represents how many carnations need to be sold to break even.

c. No; because the solution is not a whole number of carnations; 11 carnations.

18a. Hats Off: $c = 5h + 50$; Top Stuff: $c = 6h + 25$

b. 25 hats; $175; for graph, see p. A23.

State Resources

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GUIDED PRACTICE

1. **Vocabulary** Describe a *solution of a system of linear equations.* an ordered pair that satisfies both equations

SEE EXAMPLE 1 p. 383

Tell whether the ordered pair is a solution of the given system.

2. $(2, -2)$; $\begin{cases} 3x + y = 4 \\ x - 3y = -4 \end{cases}$ no

3. $(3, -1)$; $\begin{cases} x - 2y = 5 \\ 2x - y = 7 \end{cases}$ yes

4. $(-1, 5)$; $\begin{cases} -x + y = 6 \\ 2x + 3y = 13 \end{cases}$ yes

SEE EXAMPLE 2 p. 384

Solve each system by graphing. Check your answer.

5. $\begin{cases} y = \frac{1}{2}x \\ y = -x + 3 \end{cases}$ $(2, 1)$

6. $\begin{cases} y = x - 2 \\ 2x + y = 1 \end{cases}$ $(1, -1)$

7. $\begin{cases} -2x - 1 = y \\ x + y = 3 \end{cases}$ $(-4, 7)$

SEE EXAMPLE 3 p. 385

8. To deliver mulch, Lawn and Garden charges $30 per cubic yard of mulch plus a $30 delivery fee. Yard Depot charges $25 per cubic yard of mulch plus a $55 delivery fee. For how many cubic yards will the cost be the same? What will that cost be? 5 yd^3; $180

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
9–11	1
12–15	2
16	3

Extra Practice
Skills Practice p. S14
Application Practice p. S33

Tell whether the ordered pair is a solution of the given system.

9. $(1, -4)$; $\begin{cases} x - 2y = 8 \\ 4x - y = 8 \end{cases}$ no

10. $(-2, 1)$; $\begin{cases} 2x - 3y = -7 \\ 3x + y = -5 \end{cases}$ yes

11. $(5, 2)$; $\begin{cases} 2x + y = 12 \\ -3y - x = -11 \end{cases}$ no

Solve each system by graphing. Check your answer.

12. $\begin{cases} y = \frac{1}{2}x + 2 \\ y = -x - 1 \end{cases}$ $(-2, 1)$

13. $\begin{cases} y = x \\ y = -x + 6 \end{cases}$ $(3, 3)$

14. $\begin{cases} -2x - 1 = y \\ x = -y + 3 \end{cases}$ $(-4, 7)$

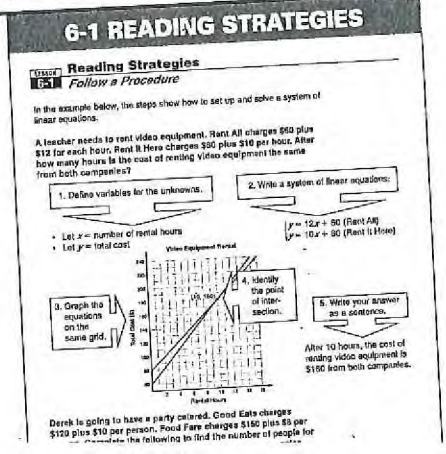
15. $\begin{cases} x + y = 2 \\ y = x - 4 \end{cases}$ $(3, -1)$

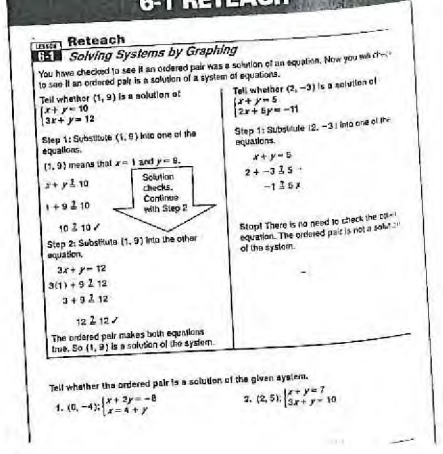
16. **Multi-Step** Angelo runs 7 miles per week and increases his distance by 1 mile each week. Marc runs 4 miles per week and increases his distance by 2 miles each week. In how many weeks will Angelo and Marc be running the same distance? What will that distance be? 3 weeks; 10 mi

17. **School** The school band sells carnations on Valentine's Day for $2 each. They buy the carnations from a florist for $0.50 each, plus a $16 delivery charge.
   a. Write a system of equations to describe the situation.
   b. Graph the system. What does the solution represent?
   c. Explain whether the solution shown on the graph makes sense in this situation. If not, give a reasonable solution.

MULTI-STEP TEST PREP

18. This problem will prepare you for the Multi-Step Test Prep on page 412.
   a. The Warrior baseball team is selling hats as a fund-raiser. They contacted two companies. Hats Off charges a $50 design fee and $5 per hat. Top Stuff charges a $25 design fee and $6 per hat. Write an equation for each company's pricing.
   b. Graph the system of equations from part a. For how many hats will the cost be the same? What is that cost?
   c. Explain when it is cheaper for the baseball team to use Top Stuff and when it is cheaper to use Hats Off. fewer than 25 hats: Top Stuff; more than 25 hats: Hats Off





19. $\begin{cases} y = 4.7x + 2.1 \\ y = 1.6x - 5.4 \end{cases}$ $(-2.4, -9.3)$

20. $\begin{cases} 4.8x + 0.6y = 4 \\ y = -3.2x + 2.7 \end{cases}$ $(0.8, 0.1)$

21. $\begin{cases} y = \frac{5}{4}x - \frac{2}{3} \\ \frac{8}{3}x + y = \frac{5}{9} \end{cases}$ $(0.3, -0.3)$

22. $\begin{cases} y = 6.9x + 12.4 \\ y = -4.1x - 5.3 \end{cases}$ $(-1.6, 1.3)$

23. Landscaping The gardeners at Middleton Place Gardens want to plant a total of 45 white and pink hydrangeas in one flower bed. In another flower bed, they want to plant 120 hydrangeas. In this bed, they want 2 times the number of white hydrangeas and 3 times the number of pink hydrangeas as in the first bed. Use a system of equations to find how many white and how many pink hydrangeas the gardeners should buy altogether. **45 white; 120 pink**

24. Fitness Rusty burns 5 Calories per minute swimming and 11 Calories per minute jogging. In the morning, Rusty burns 200 Calories walking and swims for x minutes. In the afternoon, Rusty will jog for x minutes. How many minutes must he jog to burn at least as many Calories y in the afternoon as he did in the morning? Round your answer up to the next whole number of minutes. **34 min**

25. A tree that is 2 feet tall is growing at a rate of 1 foot per year. A 6-foot tall tree is growing at a rate of 0.5 foot per year. In how many years will the trees be the same height? **8 yr**

26. Critical Thinking Write a real-world situation that could be represented by the system $\begin{cases} y = 3x + 10 \\ y = 5x + 20 \end{cases}$.

27. Write About It When you graph a system of linear equations, why does the intersection of the two lines represent the solution of the system? **Every point on a line satisfies the related linear equation. A point that is on both lines (the intersection point) satisfies both equations.**

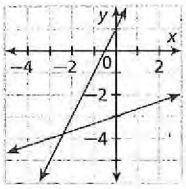
TEST PREP

28. Taxi company A charges \$4 plus \$0.50 per mile. Taxi company B charges \$5 plus \$0.25 per mile. Which system best represents this problem?

Ⓐ $\begin{cases} y = 4x + 0.5 \\ y = 5x + 0.25 \end{cases}$ Ⓒ $\begin{cases} y = -4x + 0.5 \\ y = -5x + 0.25 \end{cases}$

Ⓑ $\begin{cases} y = 0.5x + 4 \\ y = 0.25x + 5 \end{cases}$ Ⓓ $\begin{cases} y = -0.5x + 4 \\ y = -0.25x + 5 \end{cases}$

29. Which system of equations represents the given graph?



Ⓕ $\begin{cases} y = 2x - 1 \\ y = \frac{1}{3}x + 3 \end{cases}$ Ⓗ $\begin{cases} y = 2x + 1 \\ y = \frac{1}{3}x - 3 \end{cases}$

Ⓖ $\begin{cases} y = -2x + 1 \\ y = 2x - 3 \end{cases}$ Ⓙ $\begin{cases} y = -2x - 1 \\ y = 3x - 3 \end{cases}$

30. Gridded Response Which value of b will make the system $y = 2x + 2$ and $y = 2.5x + b$ intersect at the point $(2, 6)$? **1**

LINK

Landscaping

Middleton Place Gardens, South Carolina, are the United States' oldest landscaped gardens. The gardens were established in 1741 and opened to the public in the 1920s.

In **Exercises 5–7** and **12–15**, students may get the wrong answer because of inaccurate graphs. Caution students to use a straightedge when drawing lines.

MULTI-STEP TEST PREP **Exercise 18** involves writing a system of equations to compare pricing options. This exercise prepares students for the Multi-Step Test Prep on page 412.

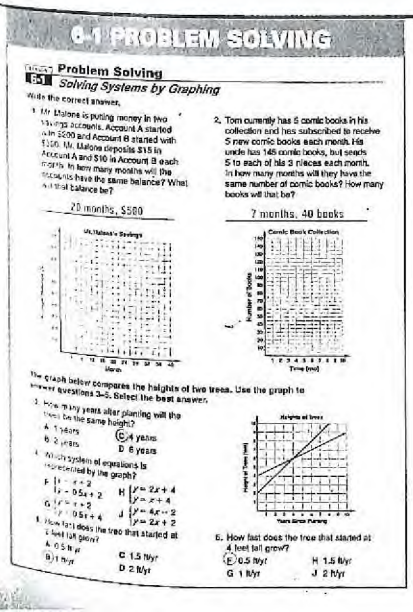
Teaching Tip **Inclusion** In **Exercise 23**, encourage students to check that they answered the question asked in the problem.

TEST PREP DOCTOR In **Exercise 28**, choices **C** and **D** can be eliminated because the situation in the problem requires boundary lines with positive slopes.

Choices **G** and **J** can be eliminated in **Exercise 29** because the graph shows two lines with positive slopes.

Answers

26. Possible answer: Store A rents carpet cleaners for a fee of \$10, plus \$3 per day. Store B rents carpet cleaners for a fee of \$20, plus \$5 per day. For how many days rental will the costs be the same?


6-1 PROBLEM SOLVING

Problem Solving

Solving Systems by Graphing

Write the correct answer.

1. Mr. Malone is putting money in two savings accounts. Account A started with \$200 and Account B started with \$100. Mr. Malone deposits \$15 in Account A and \$10 in Account B each month. In how many months will the accounts have the same balance? What will that balance be?

20 months, \$500

2. Tom currently has 5 comic books in his collection and has subscribed to receive 5 new comic books each month. His uncle has 145 comic books, but sends 5 to each of his 3 nieces each month. In how many months will they have the same number of comic books? How many books will that be?

7 months, 40 books

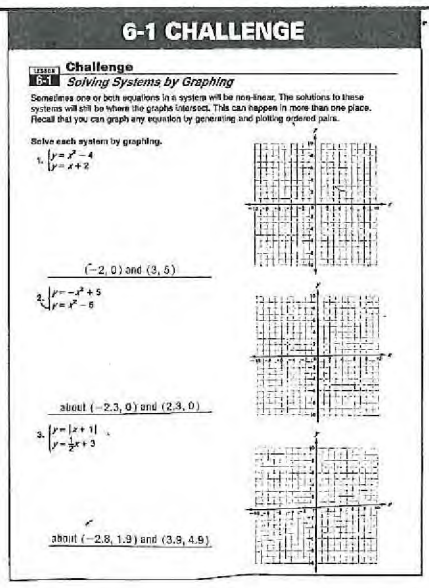
The graph below compares the heights of two trees. Use the graph to answer questions 3–5. Select the best answer.

3. How many years after planting will the trees be the same height?
A 1 years
B 2 years
C 4 years
D 6 years

4. Which system of equations is represented by the graph?
F $\begin{cases} y = x + 2 \\ y = 0.5x + 2 \end{cases}$
G $\begin{cases} y = x + 2 \\ y = x + 4 \end{cases}$
H $\begin{cases} y = 2x + 4 \\ y = x + 4 \end{cases}$
J $\begin{cases} y = 4x - 2 \\ y = 2x + 2 \end{cases}$

5. How fast does the tree that started at 2 feet tall grow?
A 0.5 ft/yr
B 1 ft/yr
C 1.5 ft/yr
D 2 ft/yr

6. How fast does the tree that started at 4 feet tall grow?
E 0.5 ft/yr
G 1 ft/yr
H 1.5 ft/yr
J 2 ft/yr


6-1 CHALLENGE

Challenge

Solving Systems by Graphing

Sometimes one or both equations in a system will be non-linear. The solutions to these systems will still be where the graphs intersect. This can happen in more than one place. Recall that you can graph any equation by generating and plotting ordered pairs.

Solve each system by graphing.

1. $\begin{cases} y = x^2 - 4 \\ y = x + 2 \end{cases}$

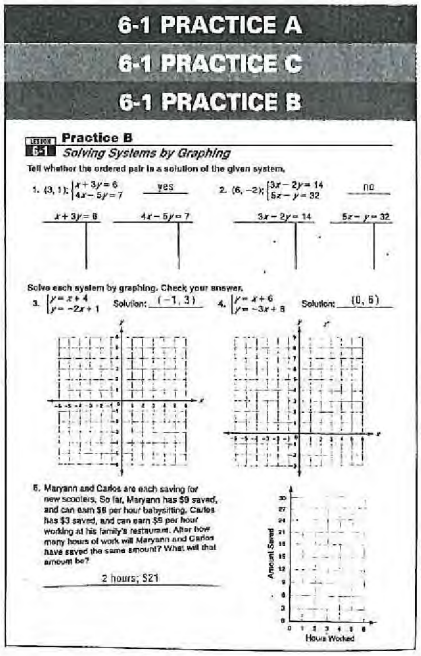
(−2, 0) and (3, 5)

2. $\begin{cases} y = -x^2 + 5 \\ y = x^2 - 5 \end{cases}$

about (−2.3, 0) and (2.3, 0)

3. $\begin{cases} y = |x + 1| \\ y = \frac{1}{2}x + 3 \end{cases}$

about (−2.8, 1.9) and (3.9, 4.9)


6-1 PRACTICE A

6-1 PRACTICE C

6-1 PRACTICE B

Practice B

Solving Systems by Graphing

Tell whether the ordered pair is a solution of the given system.

1. (3, 1); $\begin{cases} x + 3y = 6 \\ 4x - 5y = 7 \end{cases}$ yes

2. (6, −2); $\begin{cases} 3x - 2y = 14 \\ 5x - y = 32 \end{cases}$ no

$x + 3y = 6$ $4x - 5y = 7$ $3x - 2y = 14$ $5x - y = 32$

Solve each system by graphing. Check your answer.

3. $\begin{cases} y = x + 4 \\ y = -2x + 1 \end{cases}$ Solution: (−1, 3)

4. $\begin{cases} y = x + 6 \\ y = -3x + 8 \end{cases}$ Solution: (0, 6)

5. Maryann and Carlos are each saving for new scooters. So far, Maryann has \$9 saved, and can earn \$8 per hour babysitting. Carlos has \$3 saved, and can earn \$6 per hour working at his family's restaurant. After how many hours of work will Maryann and Carlos have saved the same amount? What will that amount be?

2 hours; \$21

Amount Saved

Hours Worked